APPENDIX III

FINITE STRAIN EFFECT

The work in Chapters III, IV, and V assumed the conventional magnetoelastic theory of Becker and Doring. It has been shown by Brown[5] that this theory, which assumes infinitesimal strain from the start, is inconsistent in small orders of strain. This inconsistency is normally of no consequence in magnetostrictive processes due to the extremely small strains involved (the order of 10^{-5}). In the present effect, strains of 10^{-2} or larger are realized. For this reason, it is necessary that the effect of finite strain be considered.

III.1. The Finite Strain Tensor

In the spirit of Thurston,[27] the deformation gradient for uniaxial strain colinear with the unit vector $\hat{n}$ is

$$\frac{\partial x_i}{\partial a_j} = e n_i n_j + \delta_{ij} \qquad \text{(III.1)}$$

where

$$e = \frac{\rho_o}{\rho} - 1$$

is the extension in the direction of uniaxial strain, the x_i are the Eulerian or spacial coordinates, and the a_j are the Lagrangian or material coordinates. From the Lagrangian definition of finite strain,

$$E_{ij} = \frac{1}{2}\left(\frac{\partial x_k}{\partial a_i}\frac{\partial x_k}{\partial a_j} - \delta_{ij}\right),$$

one obtains the finite uniaxial strain tensor

$$E_{ij} = \left(e + \frac{e^2}{2}\right) n_i n_j. \quad \text{(III.2)}$$

III.2. Finite Strain Correction to Interacting Grain Theory

It was shown in Section 3.2 that crystal anisotropy energy does not contribute in the first order to the shock induced anisotropy effect under conventional magnetoelastic theory. This does not follow from finite strain theory. From Equation (2.13),

$$\mathcal{E}_K = K_1(\alpha_1^{*2}\alpha_2^{*2} + \alpha_2^{*2}\alpha_3^{*2} + \alpha_3^{*2}\alpha_1^{*2}), \quad \text{(III.3)}$$

where

$$\alpha_i^* = \frac{\partial x_j}{\partial a_i}\alpha_j$$

$$= (en_i n_j + \delta_{ij})\alpha_j. \quad \text{(III.4)}$$

Substituting into Equation (III.3) gives terms, to first order in e, of the form

$$\alpha_1^{*2}\alpha_2^{*2} = \alpha_1^2\alpha_2^2 + 2e\alpha_1^2\alpha_2(n_2^2\alpha_2 + n_2n_3\alpha_3 + n_2n_1\alpha_1) +$$

$$2e\alpha_1\alpha_2^2(n_1^2\alpha_1 + n_1n_2\alpha_2 + n_1n_2\alpha_3) + \theta(e^2) + \ldots$$

The other terms follow by permuting indices. Collecting terms, using